AP Calculus BC
2000 Student Samples

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CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

R_1

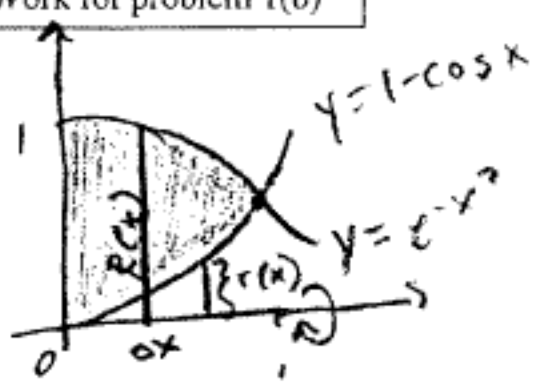
A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$A = \int_0^{.94194408} (e^{-x^2}) - (1 - \cos x)\, dx$$

$$A \approx .5907 \text{ units}^2$$

Work for problem 1(b)



$$V = \pi \int_0^{.94194408} R^2(x) - r^2(x)\, dx$$

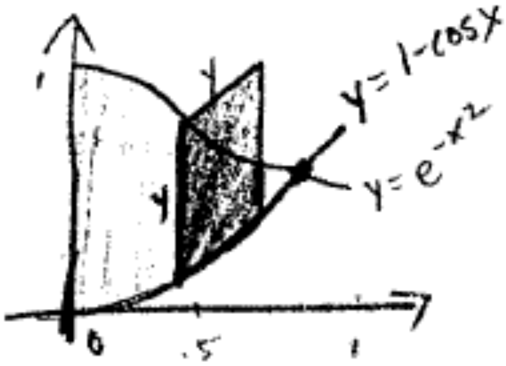
$$V = \pi \int_0^{.94194408} (e^{-x^2})^2 - (1 - \cos x)^2\, dx$$

$$V \approx 1.7466 \text{ units}^3$$

Continue problem 1 on page 5.

R_2

Work for problem 1(c)



$$V = \int \left[e^{-x^2} - (1-\cos x)\right]^2 \, dx$$

$y = e^{-x^2} - (1-\cos x)$

$y^2 = \left[e^{-x^2} - (1-\cos x)\right]^2$

$V \approx .4611$ units^3

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

T1

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

To find the right hand limit of integration I graphed the two functions and then used the intersect function on my calculator.

$$A = \int_0^{.9419} e^{-x^2} - (1 - \cos x) = \boxed{.591}$$

Work for problem 1(b)

$$V = \pi \int_0^{.9419} (e^{-x^2})^2 - (1 - \cos x)^2 = \boxed{.556\pi \text{ or } 1.75}$$

Continue problem 1 on page 5.

T_2

Work for problem 1(c)

$$V = \int_0^{.9419} \left(e^{-x^2} - (1-\cos x)\right)^2 = \boxed{.4223}$$

$A = S^2$

$S = e^{-x^2} - (1 - \cos x)$

GO ON TO THE NEXT PAGE.

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

W1

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$A = \int_0^1 e^{-x^2} - (1-\cos x)\, dx$

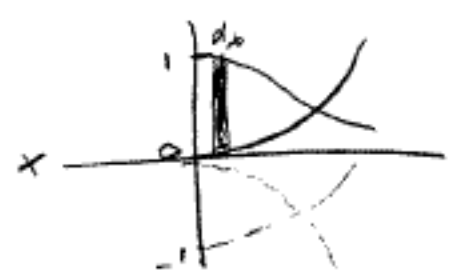
$A = 0.588\,29\ u^2$

Work for problem 1(b)

revolution about x-axis

$V = \pi \int_0^1 \left[e^{-x^2} - (1-\cos x)\right]^2 dx$

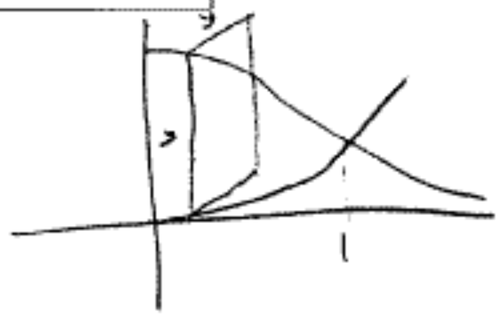
$V = 1.44899\, u^3$



Continue problem 1 on page 5.

W_2

Work for problem 1(c)



$V = \int_0^1 \left[e^{-x^2} - (1-\cos x)\right]^2 dx$

$V = 0.46123 \ u^3$

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A₁

Work for problem 2(a)

Runner A

$m = \frac{10-0}{3-0} = \frac{10}{3}$

$v - 10 = \frac{10}{3}(t-3)$

$v(t) = \frac{10}{3}t$

$v(2) = \frac{10}{3}(2)$

$= 6.667 \text{ m/s}$

Runner B

$v(t) = \frac{24t}{2t+3}$

$v(2) = \frac{24(2)}{2(2)+3}$

$= 6.857 \text{ m/s}$

Work for problem 2(b)

Runner A

$v(t) = \frac{10}{3}t$

$a(t) = v'(t) = \frac{10}{3}$

$a(2) = 3.333 \text{ m/s}^2$

Runner B

$v(t) = \frac{24t}{2t+3}$

$a(t) = v'(t) = \frac{(2t+3)(24) - (2)(24t)}{(2t+3)^2}$

$= \frac{48t + 72 - 48t}{(2t+3)^2}$

$= \frac{72}{(2t+3)^2}$

$a(2) = \frac{72}{(2(2)+3)^2}$

$= \frac{72}{49} = 1.469 \text{ m/s}^2$

Continue problem 2 on page 7.

A₂

Work for problem 2(c)

Runner A

Total Distance $= \int_0^{10} v(t)\,dt$

$= \int_0^3 \left(\frac{10}{3}t\right)dt + \int_3^{10} (10)\,dt$

$= 15_m + 70_m$

$= 85\text{ m}$

Runner B

Total Distance $= \int_0^{10} v(t)\,dt$

$= \int_0^{10} \left(\frac{24t}{2t+3}\right)dt$

$= 83.336\text{ m}$

C1

Work for problem 2(a)

a. Velocity of Runner A =
(0,0) (3,10)

$\frac{10-0}{3-10} = 10/3 = m$

(3,10) (2,y) =

$\frac{y-10}{2-3} = 10/3$

= 6.67 m/s

velocity of runner b

b. $v(t) = \frac{24t}{2t+3}$

$v(2) = \frac{(24)(2)}{2(2)+3}$

= 6.857 m/s

Work for problem 2(b)

a Runner A = ?

v/t = a

V_a = 6.67 m/s
t = 2 s

$\frac{6.67}{2}$ = 3.33 m/s^2
acceleration of runner A

a of runner b = ?

a = v/t

v = 6.857 m/s

t = 2s

$\frac{6.857}{2}$ = 3.4285

3.4285 m/s^2
acceleration of runner B

Continue problem 2 on page 7.

C_2

Work for problem 2(c)

Runner A

$d = vt$

area under curve

from 0, 10

$= \frac{1}{2}(3 \cdot 10) + 7(10)$

85 m

Runner A

Runner B

$\int v(t) = d(t)$

$\int_0^{10} \frac{24t}{2t+3} =$

= 83.336 m

Runner B

=

GO ON TO THE NEXT PAGE.

F₁

Work for problem 2(a)

Runner A graph: v(t) +

Runner B $v(t) = \frac{24t}{2t+3}$

Runner A

from the graph $v(2) = 7$ m/s

Runner B

$v(2) = \frac{24(2)}{2(2)+3}$

$= \frac{48}{7}$

$= 6.85$ m/s

Work for problem 2(b)

Runner A

(2,7) & (3,10)

acceleration at $t=2$ $= \frac{10-7}{3-2}$

$= 3$ m/s^2

Runner B

$v(t) = \frac{24t}{2t+3}$

$a(t) = \frac{(24)(2t+3)-(24t)(2)}{(2t+3)^2}$

$= \frac{48t+72-48t}{(2t+3)^2}$

$a(2) = \frac{72}{(2(2)+3)^2}$

$= \frac{72}{(7)^2}$

$= 1.47$ m/s^2

Continue problem 2 on page 7.

F_2

Work for problem 2(c)

Total distance covered by Runner A = area under the v-t graph

$= \frac{1}{2}(\cancel{10}5)(3) + (7)(10)$

$= 15 + 70$

$= 85\ m$

Total distance covered by Runner B $= \int_0^{10} v(t)\,dt$

$= \int_0^{10} \frac{24t}{2t+3}\,dt$

$= 83.3\ m$

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Work for problem 3(a)

$$f^{0}(5) = \frac{(-1)^{0}\,0!}{2^{0}(0+2)} = \frac{1}{2}$$

$$f'(5) = \frac{(-1)\cdot 1}{2(3)} = -\frac{1}{6}$$

$$f^{2}(5) = \frac{(-1)^{2}\,2!}{4(4)} = \frac{1}{8}$$

$$f^{3}(5) = \frac{(-1)\,3!}{8(5)} = \frac{-6}{40} = -\frac{3}{20}$$

$$P_3 = \frac{1}{2} - \frac{1}{6}\frac{(x-5)}{1!} + \frac{1}{8}\frac{(x-5)^2}{2!} - \frac{3}{20}\frac{(x-5)^3}{3!}$$

$$\boxed{P_3 = \frac{1}{2} - \frac{1}{6}(x-5) + \frac{1}{16}(x-5)^2 - \frac{1}{40}(x-5)^3}$$

Work for problem 3(b)

$$\sum_{n=0}^{\infty} \frac{(-1)^n n!}{2^n(n+2)} \cdot \frac{(x-5)^n}{n!}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x-5)^n}{2^n(n+2)}$$

Ratio Test:

$$\lim_{n\to\infty} \left| \frac{(x-5)^n(x-5)}{2^n\cdot 2\,(n+3)} \cdot \frac{2^n(n+2)}{(x-5)^n} \right| < 1$$

$$\lim_{n\to\infty} \left| \frac{n+2}{2n+3} \cdot (x-5) \right| < 1$$

$$\frac{1}{2}|x-5| < 1$$

$$|x-5| < 2$$

Continue problem 3 on page 9.

$\boxed{\text{radius of convergence} = 2}$

Work for problem 3(c)

Remainder left off is less than first term

Alternating series, so $\longrightarrow R_n \leq |a_{n+1}|$ (R = error)

$R_6 \leq \left| \frac{(6-5)^7}{2^7(7+2)} \right|$

$R_6 \leq 8.68055556 \times 10^{-4}$

$R_6 \leq .00086805556$

and

$.0008680556 \leq .001$

so the sixth degree Taylor polynomial approximates f with an error less than $\frac{1}{1000}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

 C_1

Work for problem 3(a)

$$\frac{f^n(5)\cdot(x-5)^n}{n!} = \frac{(-1)^n\cdot(x-5)^n}{2^n(n+2)}$$

$$\frac{1}{2} - \frac{(x-5)}{6} + \frac{(x-5)^2}{16} - \frac{(x-5)^3}{40}$$

Work for problem 3(b)

ratio test

$$\frac{(-1)^{n+1}\cdot(x-5)^{n+1}}{2^{n+1}(n+3)} \cdot \frac{2^n(n+2)}{(-1)^n(x-5)^n} = \lim_{n\to\infty}\left|\frac{(x-5)(n+2)}{2(n+3)}\right| = \frac{1}{2}$$

$$\left|\frac{x-5}{2}\right| < 1$$

$$-2 < |x-5| < 2$$

$$\boxed{3 < x \leq 7}$$

Continue problem 3 on page 9.

C_2

Work for problem 3(c)

6th degree term

$$\frac{(x-5)^6}{512} \Rightarrow \frac{1}{512}$$

$x=6$

next term

$$\frac{(x-5)^7}{1152} = \frac{1}{1152}$$

$x=6$

a sixth degree Taylor polynomial will aproximate $f(6)$ with an error less than $1/1000$ because every term after it has a value of less than $1/1000$ for $f(6)$

END OF PART A OF SECTION II

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Work for problem 3(a)

centered at $x=5$ so $c=5$

$$P(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!}$$

$$P(x) = 5 + \frac{(-1)^1(1!)}{2^{(1)}(1+2)}(x-c) + \frac{(-1)^{(2)}(2)!}{2^{(2)}((2)+2)} \cdot \frac{(x-c)^2}{2!} + \frac{(-1)^{(3)}(3)!}{2^{(3)}((3)+2)} \frac{(x-c)^3}{3!} + \ldots$$

$$P(x) = 5 - \frac{(x-5)}{6} + \frac{(x-5)^2}{16} - \frac{(x-5)^3}{40} + \ldots$$

n^{th} term given by $\frac{(-1)^n n!}{2^n(n+2)} \cdot \frac{(x-5)^n}{n!} = \frac{(-1)^n (x-5)^n}{2^n(n+2)}$

Work for problem 3(b)

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-5)^{n+1}}{2^{n+1}(n+3)} \cdot \frac{2^n(n+2)}{(-1)^n(x-5)^n} \right| = \lim_{n\to\infty} \left| \frac{(x-5)(n+2)}{2(n+3)} \right| = \left|\frac{x-5}{2}\right| \lim_{n\to\infty} \frac{n+2}{n+3} = \left|\frac{x-5}{2}\right|$$

$\left|\frac{x-5}{2}\right| < 1$ by ratio test, if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, series converges

$|x-5| < 2$

$x-5=2$ or $5-x=2$

$x=7$ or $x=3$

$3 < x < 7$

$\therefore$ the radius of convergence for this series is $7-3$ or 4.

Continue problem 3 on page 9.

 E_2

Work for problem 3(c)

n^{th} term is $\dfrac{(-1)^n (x-5)^n}{2^n (n+2)}$

because $P(x)$ is an alternating series, the alternating series remainder theorem guarentees that error for n terms will be less than or equal to the |n+1 term|

for a 6^{th} degree polynomial, the error is $\leq |7^{th}$ term$|$

$$\text{error} \leq \left| \frac{(-1)^7 (x-5)^7}{2^7 (n+2)} \right| \quad x = 6$$

$$\text{error} \leq \left| \frac{(-1)^7 (6-5)^7}{2^7 (6+2)} \right|$$

$$\text{error} \leq \frac{1}{1024}$$

$$\therefore \frac{1}{1024} < \frac{1}{1000}$$

$\therefore$ the error for a 6^{th} degree polynomial is less than $\frac{1}{1000}$

END OF PART A OF SECTION II

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A1

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$\vec{v} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \left\langle 1 - \frac{1}{t^2}, 2 + \frac{1}{t^2} \right\rangle$$

$$\vec{a} = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle = \left\langle 2t^{-3}, -2t^{-3} \right\rangle$$

$$\Rightarrow \vec{a}(3) = \left\langle \frac{2}{27}, \frac{-2}{27} \right\rangle$$

Work for problem 4(b)

$$x(t) = \int \left(\frac{dx}{dt}\right) dt = \int (1 - t^{-2})\, dt = t + t^{-1} + c$$

$$x(1) = 2 + c = 2 \Rightarrow c = 0$$

$$y(t) = \int \left(\frac{dy}{dt}\right) dt = \int (2 + t^{-2})\, dt = 2t - t^{-1} + c$$

$$y(1) = 1 + c = 6 \Rightarrow c = 5$$

$$\Rightarrow (x(t), y(t)) = (t + t^{-1}, 2t - t^{-1} + 5)$$

$$\therefore (x(3), y(3)) = \left(\frac{10}{3}, \frac{32}{3} \right)$$

Continue problem 4 on page 11.

Work for problem 4(c)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 + t^{-2}}{1 - t^{-2}} = \frac{2t^2 + 1}{t^2 - 1} = 8$$

$$\Leftrightarrow \quad 6t^2 = 9$$

$$\Leftrightarrow \quad t = \sqrt{\tfrac{3}{2}} \quad , \quad t > 0$$

Work for problem 4(d)

$$\lim_{t \to \infty} \left(\frac{dy}{dx}\right) = \lim_{t \to \infty} \left(\frac{2t^2 + 1}{t^2 - 1}\right)$$

; both numerator & denominator are undefined, so by L'Hopital's Rule,

$$= \lim_{t \to \infty} \left(\frac{4t}{2t}\right) = \lim_{t \to \infty} 2 = 2$$

∴ the slope of the line ~~that~~ $y(x)$ approaches is 2

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

C_1

Work for problem 4(a)

$\text{velocity} = (1-t^{-2}, 2+t^{-2})$

$\text{acc} = (2t^{-3}, -2t^{-3})$

$\left(\frac{2}{t^3}, -\frac{2}{t^3}\right)$ at $t=3$

$\boxed{\left(\frac{2}{27}, \frac{-2}{27}\right)}$

Work for problem 4(b)

$\text{velocity} = (1-t^{-2}, 2+t^{-2})$

$t=1$

pos $(2,6)$

$x(t)$

$\int(1-t^{-2})\,dt$

$t+t^{-1}+C$

$2 = 1+1+C$

$C=0$

$x(t) = t + \frac{1}{t}$

$y(t)$

$\int(2+t^{-2})\,dt$

$2t - t^{-1} + C$

$6 = 2 - 1 + C$

$6 = 1 + C$

$C=5$

$y(t) = 2t + \frac{1}{t} + 5$

$6 + \frac{1}{3} + 5 =$

at $t=3$

position $\boxed{\left(\frac{4}{3}, 11\frac{1}{3}\right)}$

Continue problem 4 on page 11.

C2

Work for problem 4(c)

$\frac{dx}{dt} = 1 - t^{-2}$ $\frac{dy}{dt}$ $2 + t^{-2}$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{2+t^{-2}}{1-t^{-2}} \qquad \frac{2+t^{-2}}{1-t^{-2}} = 8$$

$$2 + t^{-2} = 8 - 8t^{-2}$$

$$9t^{-2} = 6$$

$$\frac{9}{t^2} = 6 \qquad 6t^2 = 9$$

$$t^2 = \frac{9}{6}$$

$$t = \sqrt{\frac{9}{6}} = \boxed{\frac{3}{\sqrt{6}}}$$

Work for problem 4(d)

$$\lim_{t\to\infty} \frac{2+t^{-2}}{1-t^{-2}} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$

take deriv of top + bottom L'Hopital's rule

$$\lim_{t\to\infty} \frac{-2t^{-3}}{2t^{-3}} = \boxed{-1}$$

F1

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

$-t^{-2}$

$+2t^{-3}$

Work for problem 4(a)

$v(t) = \left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right) \Rightarrow v(t) = (x, y)$

$a(t) = (dx, dy)$

$a(t) = \left(\frac{1}{2t^3}, \frac{-1}{2t^3}\right)$

$a(3) = \left(\frac{1}{2(3)^3}, \frac{-1}{2(3)^3}\right)$

$a(3) = \left(\frac{1}{54}, \frac{-1}{54}\right)$

Work for problem 4(b)

$t^{-1} \to -t^{-2}$

$v(t) = \left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right) \Rightarrow v(t) = (dx, dy)$

$p(t) = (x, y)$

$p(t) = \left(t + \frac{1}{t}, 2t - \frac{1}{t}\right)$

$p(3) = \left(3\frac{1}{3}, 5\frac{2}{3}\right)$

Continue problem 4 on page 11.

F2

Work for problem 4(c)

$$\frac{dy}{dx} = 8 = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$

$$8 - \frac{8}{t^2} = 2 + \frac{1}{t^2}$$

$$6 = \frac{9}{t^2}$$

$$6t^2 = 9$$

$$t^2 = 3/2$$

$$t = \sqrt{3}/\sqrt{2}$$

Work for problem 4(d)

$$\lim_{t \to \infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$$

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A₁

Work for problem 5(a)

(a) $xy^2 - x^3y = 6$

$x(2y)(\frac{dy}{dx}) + y^2 - 3x^2y - x^3(\frac{dy}{dx}) = 0$

$(\frac{dy}{dx})2yx - (\frac{dy}{dx})x^3 = 3x^2y - y^2$

$\frac{dy}{dx}(2yx - x^3) = 3x^2y - y^2$

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2yx - x^3}$

Work for problem 5(b)

$(1)y^2 - (1)y = 6$

$y^2 - y - 6 = 0$

$(y+2)(y-3) = 0$ $(1,-2)$ and $(1,3)$

$y = -2$ or 3

at $(1,-2)$

$\frac{dy}{dx}\Big|_{(1,-2)} = \frac{3(1)(-2)-4}{2(-2)(1)-(1)} = \frac{-6-4}{-4-1} = \frac{-10}{-5} = 2$

$y = 2(x-1) - 2$

$\boxed{y = 2x - 4}$

at $(1,3)$ $\frac{dy}{dx}\Big|_{(1,3)} = \frac{3(3)-(3)^2}{2(3)-1} = \frac{9-9}{6-1} = 0$

$\boxed{y = 3}$

Continue problem 5 on page 13.

A_2

Work for problem 5(c)

(c) $\frac{dy}{dx}$ does not exist, y does exist

& $\frac{dy}{dx}$ does not change sign on either side of pt.

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ which D.N.E.

y does exist when $xy^2 - x^3y = 6$

when $2xy - x^3 = 0$

$x(2y - x^2) = 0$

$x = 0$ or $2y - x^2 = 0$

$2y = x^2$

$y = \frac{x^2}{2}$

$(0)y - (0)y = 6$

$0 \neq 6$

$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$

$\frac{x^5}{4} - \frac{2x^5}{4} = 6$

$-\frac{x^5}{4} = 6$

$x^5 = -24$

$\boxed{x = \sqrt[5]{-24}}$

GO ON TO THE NEXT PAGE.

Work for problem 5(a)

$xy^2 - (x^3y) = 6$

$(x \cdot 2yy') + (y^2) - [(x^3y') + (3x^2y \cdot)] = 0$

$2xyy' + y^2 - x^3y' - 3x^2y = 0$

$2xyy' - x^3y' = 3x^2y - y^2$

$y'(2xy - x^3) = 3x^2y - y^2$

$y' = \dfrac{3x^2y - y^2}{2xy - x^3}$

Work for problem 5(b)

$x = 1$

$xy^2 - x^3y = 6$

$(1)y^2 - (1)^3y = 6$

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2)$

$y = 3, -2$

$m = \dfrac{3x^2y - y^2}{2xy - x^3}$ for $(1, 3)$ $m = 0$

$m = \dfrac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3}$

$m = \dfrac{9 - 9}{6 - 1} = 0$

so pts are (1,3) and (1,-2)

$m = \dfrac{3x^2y - y^2}{2xy - x^3}$ at $(1,-2)$

$m = \dfrac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3}$

$= \dfrac{-6 - 4}{-4 - 1} = \dfrac{-10}{-5} =$

- so eq^n for (1,3) ($m = 0$); $y - 3 = 0(x-1)$ → $y = 3$
- eq^n for (1,-2) ($m = 2$); $y + 2 = 2(x-1)$

Continue problem 5 on page 13.

Work for problem 5(c)

tangent line = vertical when denominator of $\frac{dx}{dy}$ is 0

so $0 = 2xy - x^3$

Work for problem 5(a)

$xy^2 - x^3y = 6$

$x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1 - x^3\frac{dy}{dx} + y(-3x^2) = 0$

$2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = 3x^2y - y^2$

$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$

Work for problem 5(b)

$(1)y^2 - (1)^3y = 6$

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$\frac{1 \pm \sqrt{-1^2 - 4(1)(-6)}}{2(1)}$

$\frac{1 \pm \sqrt{25}}{1}$

$\frac{1+5}{1}$ or $\frac{1-5}{1}$

6 or -4

$\boxed{(1,6) \text{ and } (1,-4)}$

$\frac{3x^2y - y^2}{2xy - x^3} = \frac{dy}{dx}$

$\frac{3(1)^2(6) - 6^2}{2(1)(6) - (1)^3} = \frac{18 - 36}{12 - 1} = \frac{-18}{11} = \frac{dy}{dx}$

$\rightarrow y - 6 = -\frac{18}{11}(x - 1)$

$\boxed{y = -\frac{18}{11}(x - 1) + 6}$

$\frac{3x^2y - y^2}{2xy - x^3} = \frac{dy}{dx}$

$\frac{3(1)^2 \cdot 4 - 4^2}{2(1)(4) - 1^3} = \frac{12 - 16}{8 - 1} = \frac{-4}{7}$

$y - 4 = -\frac{4}{7}(x - 1)$

$\boxed{y = -\frac{4}{7}(x - 1) + 4}$

Continue problem 5 on page 13.

Work for problem 5(c)

$$2xy - x^3 = 0$$

$$2x\left(-\tfrac{4}{7}(x-1)+4\right) - x^3 = 0$$

$$2x\left(-\tfrac{4}{7}x + \tfrac{4}{7} + 4\right)$$

$$-\tfrac{8}{7}x^2 + \tfrac{8}{7}x + \tfrac{8}{1}x = 0$$

$$-\tfrac{8}{7}x^2 + \tfrac{64}{7}x = 0$$

$$\tfrac{8}{7}x\,(x+8) = 0$$

$$\boxed{\begin{array}{l} x = 0 \\ x = -8 \end{array}}$$

AP Calculus BC
2000 Student Samples

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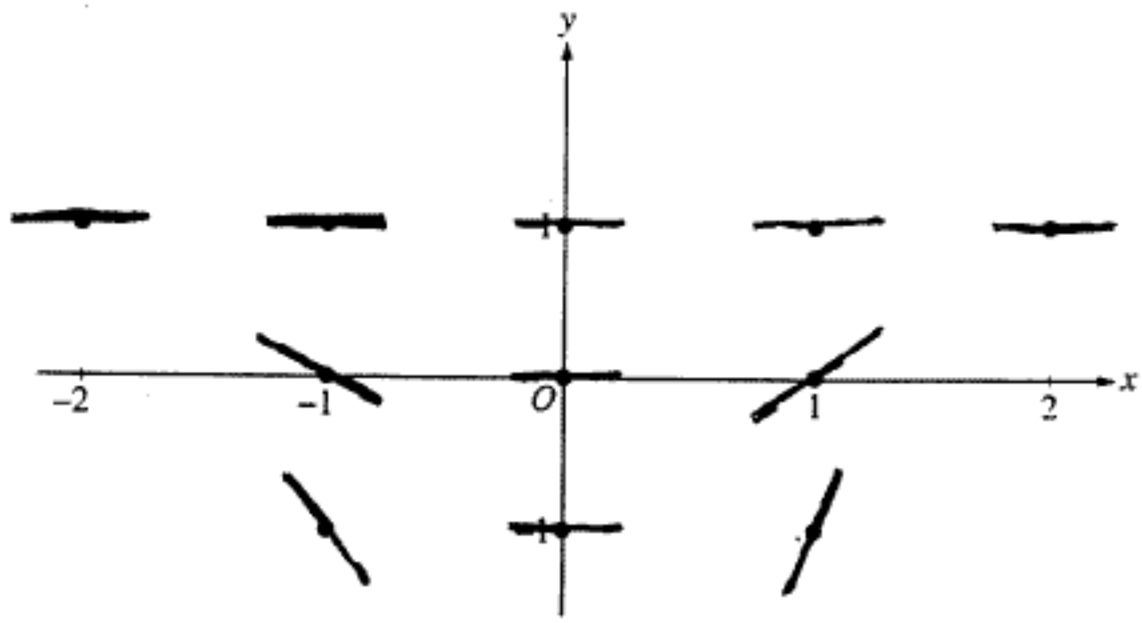
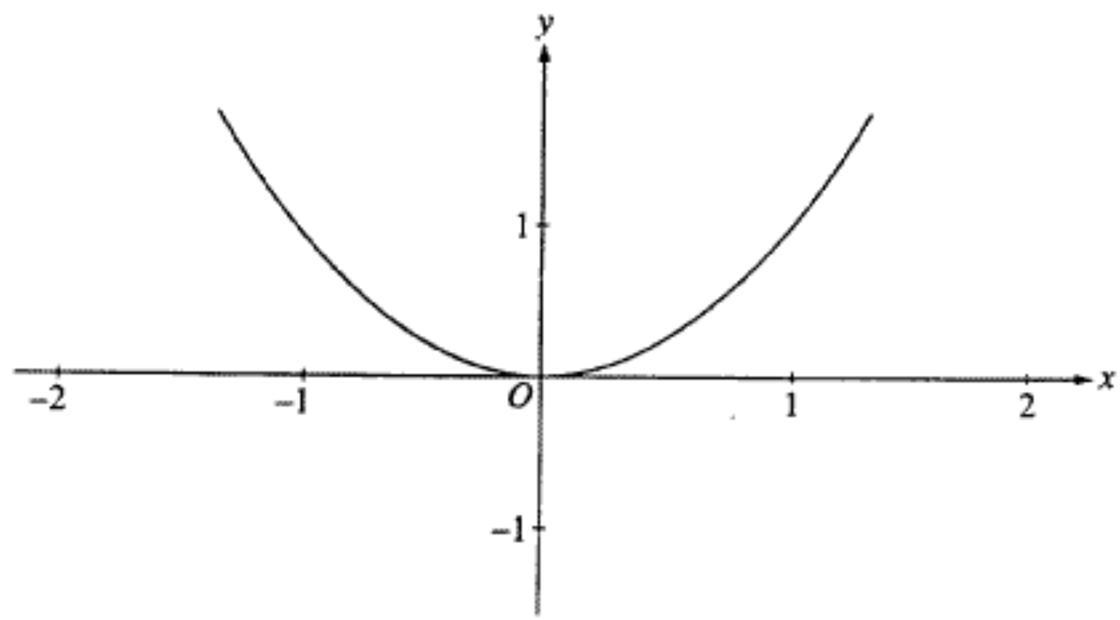
Work for problem 6(a)

at (0,0) = 0
(0,1) = 0
(0,-1) = 0
(1,0) = 1
(1,1) = 0
(1,-1) = 4
(2,1) = 0
(-1,1) = 0
(-2,1) = 0
(-1,0) = -1
(-1,-1) = -4



Work for problem 6(b)



this graph passes through the points (-1, 1) and (1,1) and has a slope whose absolute value is greater than 0. However, in the slope field, the slope at these points is 0. therefore, this graph cannot match the differential equation

Continue problem 6 on page 15.

A₂

Work for problem 6(c)

$$\frac{dy}{dx} = x(y-1)^2$$

$$\frac{dy}{(y-1)^2} = x\,dx$$

integrate: $\frac{-1}{(y-1)} = \frac{1}{2}x^2 + C$

$$\frac{-1}{(-1-1)} = \frac{1}{2}(0)^2 + C$$

$$\frac{1}{2} = 0 + c$$

$$c = \frac{1}{2}$$

$$\frac{-1}{y-1} = \frac{1}{2}x^2 + \frac{1}{2}$$

$$\frac{-1}{\frac{1}{2}(x^2+1)} = y-1$$

$$y = \frac{-2}{x^2+1} + 1$$

Work for problem 6(d)

when $x = 0$, $y = -1$

as x approaches ∞ or $-\infty$, y goes to 1

$[-1, 1\rangle$

END OF EXAMINATION

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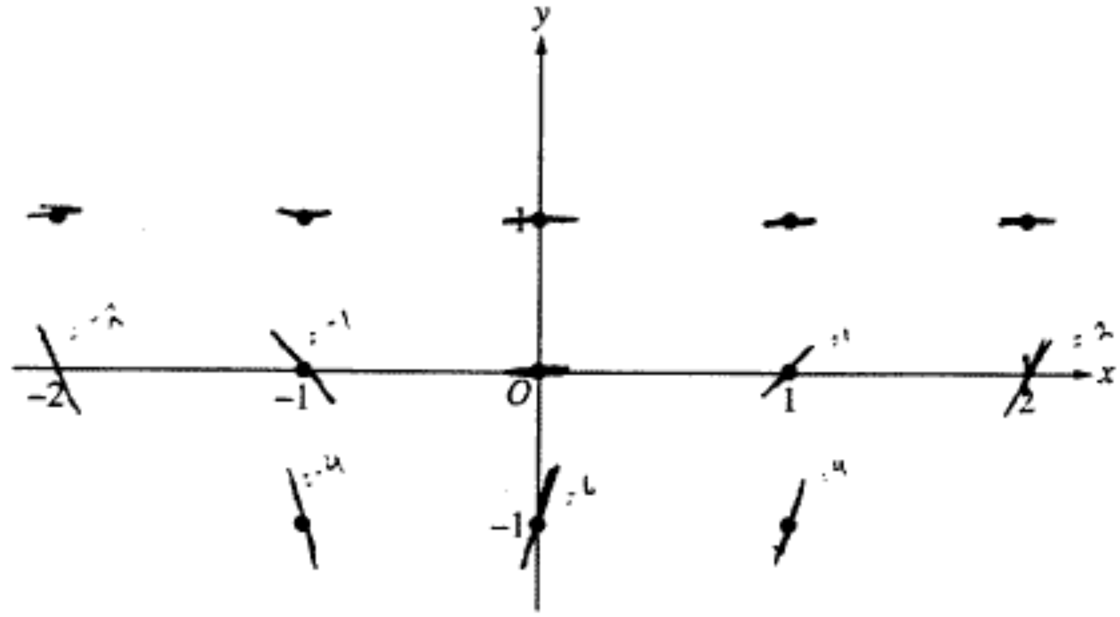
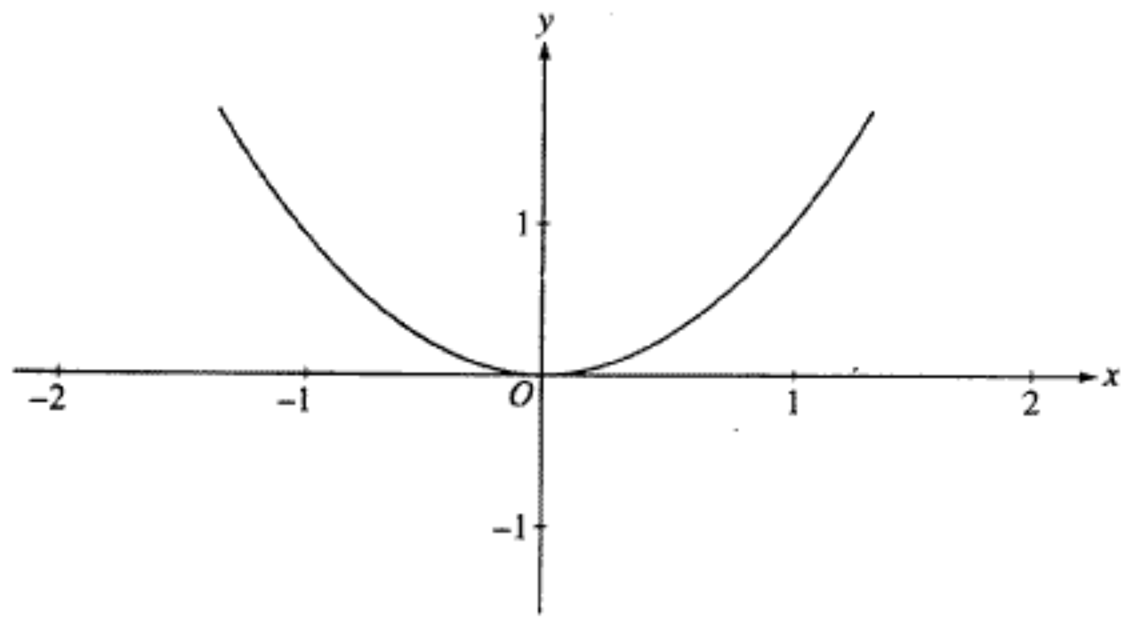
Work for problem 6(a)



Work for problem 6(b)



This graph can not be a solution because at the points (-1, 1) and (1, 1) the slope of the function must be zero. In this particular graph there is a slope not equalling 0 at these two points.

Continue problem 6 on page 15.

D_2

Work for problem 6(c)

$$\frac{dy}{(y-1)^2} = x\,dx$$

$u = y-1$

$$\int \frac{du}{u^2} = \int x\,dx$$

$$\int u^{-2}$$

$$-u^{-1} = \frac{1}{2}x^2$$

$$\frac{-1}{(y-1)} = \frac{1}{2}x^2 + c$$

$$-\frac{1}{(-1-1)} = \frac{1}{2}(0)^2 + c$$

$$-\frac{1}{-2} = c \qquad c = \frac{1}{2}$$

$$-\frac{1}{(y-1)} = \frac{1}{2}x^2 + \frac{1}{2}$$

$$-\frac{1}{y-1} = \frac{1}{2}(x^2+1)$$

$$-(y-1) = \frac{2}{x^2+1}$$

$$y = -\frac{2}{(x^2+1)} + 1$$

Work for problem 6(d)

range $: -1 < y < 1$

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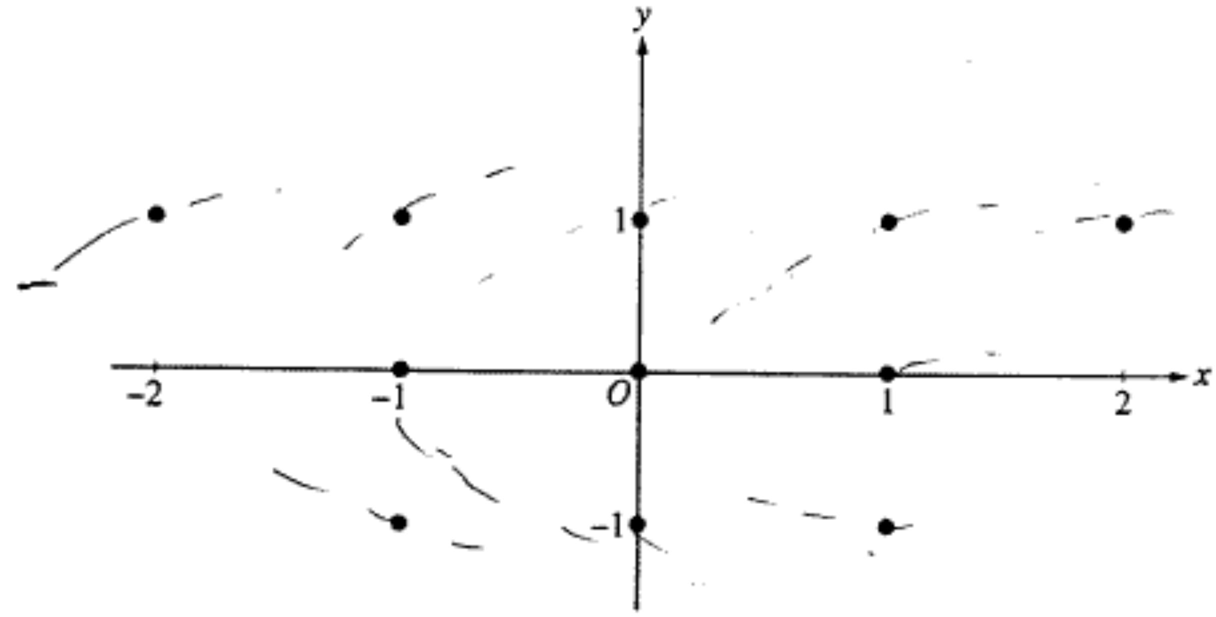
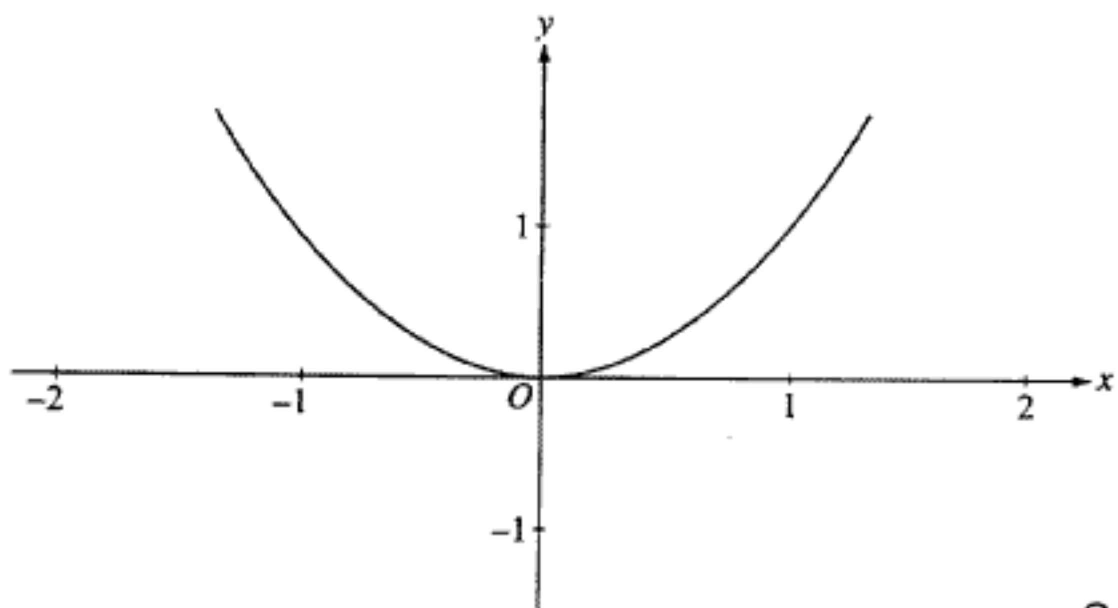
Work for problem 6(a)

$\frac{dy}{dx} = x(y-1)^2 \rightarrow y^2x - 2xy + x$



Work for problem 6(b)



Parabolas such as the one shown above have 2 values of x for each unique value y. However, the differential equation from above shows that there are more values for y than x, so the shape ~~so be~~ should ~~be~~ lie sideways.

Continue problem 6 on page 15.

Work for problem 6(c)

$f(0)=-1$

$\int \frac{dy}{(y-1)^2} = \int x\,dx \rightarrow \int \frac{dy}{y^2-2y+1} = \int x\,dx$

~~$\frac{(x-1)(x+1)}{x}$~~

$\frac{1}{2}x^2 = -(y-1)^{-1} + C$

$\frac{1}{2}x^2 + C = -\frac{1}{y-1} \rightarrow$ $(0,-1)$: $0 + C = -\frac{1}{-1-1} \rightarrow C = -\frac{1}{(-2)} = \frac{1}{2}$

$\frac{1}{2}x^2 + \frac{1}{2} = -\frac{1}{y-1} \rightarrow -\frac{1}{2}x^2 - \frac{1}{2} = \frac{1}{y-1} \rightarrow \frac{-x^2-1}{2} = \frac{1}{y-1}$

$\frac{2}{-(x^2+1)} = (y-1) \rightarrow \frac{-2}{x^2+1} + 1 = y \quad \frac{-2+x^2+1}{x^2+1} \Rightarrow \boxed{\frac{x^2-1}{x^2+1} = y}$

Work for problem 6(d)

range:

* every value of y allowed because x^2+1 can never equal 0, thus y will never be ∞

END OF EXAMINATION

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